

2.171 Problem Set 6

Assigned: Wed. Nov 1, 2006

Due: Wed. Nov 8, 2006, in class

Problem 1 Suppose we have a plant with the transfer function

$$G_p(z) = \frac{(z - 1)^3}{(z - 0.999)^3(z - 0.99)^2}$$

Assume that this plant is in the forward path of a unity-negative-feedback loop, with a controller transfer function $G_c(z)$ preceding the plant. The objective of this problem is to design a controller which stabilizes this plant with as wide a bandwidth as possible.

- a) Sketch the Bode plot of this plant. Use Matlab to confirm your sketch.
- b) Design a controller $G_c(z)$ which stabilizes this plant with as wide a bandwidth as possible. Clearly explain your design thinking. Use a Nyquist analysis to indicate the stability issues and to show that your design results in a stable system.
- c) For your chosen controller, sketch a root locus as a function of loop gain. Use Matlab to confirm your sketch.

Problem 2 FPW 7.13 a–c only.

Problem 3 This problem looks at the difficulties associated with the design in Problem 2. Specifically, we will see that the sample rate of 50 Hz specified in the problem is too low to allow a robust design. Also, we will see that the rise time spec in Problem 2 is too long to guarantee a sufficiently robust design.

- a) Plot the response of your system in x and i to an initial position error in x of 1 mm. Comment on the features of this response with respect to the designed damping and natural frequency.
- b) Sketch a Bode plot of the loop transfer function of your design in Problem 2. You should see that there are gain-increase and gain-decrease margins. By this we mean that the controlled system will go unstable for some gain-increase factor and for some gain decrease factor. Another way to say this is that the system is conditionally stable. What are these margins? What is the loop phase margin? Put this information in the form of a Nyquist plot, and clearly indicate the margins on this plot.
- c) Now, repeat your design efforts with a sample rate of 5 kHz. This will yield the possibility of acceptable control performance. Further, this time we will design to bandwidth and phase margin specifications. Develop a design which yields a gain-reduction margin of at least 4, and a gain-increase margin of at least 4. Comment on the loop bandwidth (crossover frequency) required to meet this specification. What phase margin results? Construct a Nyquist analysis for your loop design and indicate these margins on the Nyquist plot.

Problem 4 In the next problem set we will consider the control of systems with resonances well above the intended crossover frequency, and see that these cause significant challenges in achieving high loop bandwidth. In this problem, we take a brief look at the control of systems with resonances *below* the intended crossover. Such a transfer function might occur in the control of a mass mounted on a very compliant flexure. Plant A represents a free mass, and plant B represents a mass mounted on a compliant flexure. The intuition here is that the flexure has little effect above the system resonant frequency. We define the transfer functions for systems A and B as

$$H_A(z) = \frac{1}{(z-1)^2} \quad (1)$$

and

$$H_B(z) = \frac{1}{(z - Re^{j\theta})(z - Re^{-j\theta})} \quad (2)$$

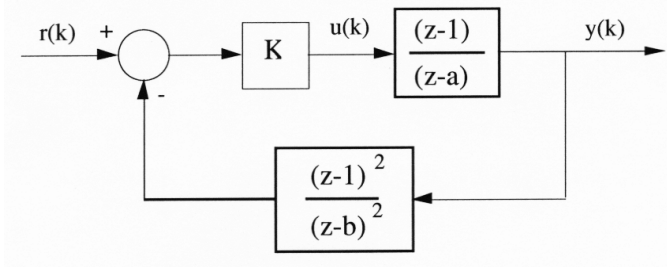
Here, the parameters have the values $R = 0.999$, and $\theta = 0.01$.

- a) Sketch Bode plots for both of these systems. Argue that the Bode plots are essentially identical for frequencies $\Omega > 0.01$ rad/sample. Explain.
- b) Use a root locus analysis to convince us that any controller with a lead network will be able to stabilize both of these plants, and that the system poles will be in nearly identical locations for both controllers once the bandwidth is above 0.01 rad/sample.
- c) Assume that we use a lead compensator of the form

$$G(z) = \frac{0.02(z - 0.95)}{(z - 0.8)} \quad (3)$$

to control plant A in a unity negative feedback loop. With these parameters, the loop crosses over at about 0.1 rad/sample. Assuming this crossover frequency, and using any reasonable approximations, calculate the phase margin of this loop.

Problem 5 Our studies so far have primarily considered loop transmissions which are essentially low-pass. However, there are also some loops which are high-pass, i.e., have no gain at DC. These types of loops also have stability issues, however, the unity magnitude crossover point now occurs at low frequencies. Consider the discrete-time feedback loop shown below.



As can be seen in the figure, this loop has a reference input $r(k)$, a control signal $u(k)$, and an output signal $y(k)$. The loop transmission has a variable gain K , three zeros at $z = 1$, one pole at $z = a$, and two poles at $z = b$. The zeros at $z = 1$ force the loop gain to be zero at DC.

- (a) For this loop, calculate the transfer functions $H_1(z) = Y(z)/R(z)$ and $H_2(z) = U(z)/R(z)$. What are the zero locations for these two transfer functions?
- (b) In this and subsequent sections, we assign the parameters $b = 0.99$ and $a = 0.9$. For these values, and with $K = 1$, sketch a Bode plot of the negative of the loop transmission for this system. For what value of frequency Ω is the phase equal to $+180$ degrees? Be sure to show how you found this frequency and include it on your Bode plot. What is the magnitude of the loop transmission at this frequency?
- (c) Sketch a root locus for this system with K as the variable loop gain. Do not bother to numerically calculate real-axis breakaway points. (Hint: The angles of approach to the zeros will help greatly in drawing the locus. Remember that the phase of the negative of the loop transmission must be an odd multiple of 180 degrees in order for a point to be on the locus. Remember also that all three poles must end up at the three zeros.) At approximately what point(s) do poles cross the unit circle? Why? Label these points on your sketch.
- (d) Using the analyses above, determine the approximate range of gain $K > 0$ for which the loop is stable. Explain your approach.
- (e) For $K = 1$, use your Bode plot from b) to develop a Nyquist analysis for this system. Note that you do not need to detour around the zeros on the unit circle, since the magnitude is zero there. Indicate how the Nyquist analysis shows the allowable range of K for stability. Be sure also to show the Γ contour on which you are evaluating the negative of the loop transmission.
- (f) In this section, replace the gain K with a transfer function $G(z)$. Choose $G(z)$ to achieve a loop crossover frequency of 0.01 rad/sample with about 45 degrees of phase margin. Clearly indicate your design thinking; also clearly state your transfer function $G(z)$ and the numerical values of its parameters. Sketch a Bode plot of the negative of the loop transmission for your design.